Dr. Wilcox. Micro 1. Problem set #8.

As some of you know, I've been translating my old problem sets from an ancient mathematical word processing software to Microsoft Word. Unfortunately, I see no way to make a proper "weak succession" operators in Microsoft Word. That is the operator I use in class to denote the relation "X is weakly preferred to Y"—the symbol that looks sort of like a "greater than or equal to" symbol. So, in this problem set, please interpret the string "$\succ\sim$" as that "weak succession" operator, so that:

"X $\succ\sim$ Y" means "X is weakly preferred to Y" (or "X is at least as good as Y");

"X $\succ$ Y" means "X is strictly preferred to Y" (or "X is better than Y"); and

"X ~ Y" means "X is indifferent to Y" (or "X and Y are equally good").

Although I am writing it this way in this problem set, you should write it the way I do in class on the board when you write up your answers.

(1) Imagine that subject *i* is presented with "repeated trials" of choosing one alternative from sets like {X,Y}, and let $P_i(X,Y)$ denote the observed proportion of trials in which subject *i* chose X from the decision set {X,Y}. This is done for all possible two-alternative sets $\{X,Y\} \subset \Omega_g$, some "grand decision set" we are interested in. Suppose we were to <u>define</u> the relation $\succ\sim_i$ according to these observed $P_i(X,Y)$ from that experiment, like so:

$X \succ\sim_i Y$ <u>means</u> $P_i(X,Y) \geq 0.5$.

This should look intuitive: It just defines the relation "X is at least as good as Y" to mean that subject *i* chose X no less frequently than Y in those choice trials of the experiment when she was presented with the choice set {X,Y}.

Assume that subjects never fail to make a choice on any trial of any decision set.

(a) What two particular things will have to be true of the proportions $P_i(X,Y)$ if we want the relational structure $\langle\Omega_g, \succ\sim_i\rangle$ for subject *i* to satisfy the formal definition of a "weak order" that I gave you in class? (Hint: Transform the two properties of a weak order so that they are written in terms of properties of $P_i(X,Y)$ rather than $\succ\sim_i$.) One of these things is always true because of the mathematical definitions of the things we call "proportions," but the other may or may not be true of the proportions $P_i(X,Y)$. Which is which?

(b) Suppose we observe $P_i(X,Y)$ for all possible pairs $\{X,Y\} \subset \Omega_g$ of alternatives from the grand decision set $\Omega_g = \{A , B , C , D \}$ for subjects *i* = 1 and 2, and these proportions are as given in the two tables (one for each subject) on the next page:

$P_1(X,Y)$		Alternative Y			
		A	B	C	D
Alternative X	A	—	0.72	0.65	0.67
	B	0.28	—	0.39	0.32
	C	0.35	0.61	—	0.40
	D	0.33	0.68	0.60	—

$P_2(X,Y)$		Alternative Y			
		A	B	C	D
Alternative X	A	—	0.70	0.60	0.25
	B	0.30	—	0.95	0.80
	C	0.40	0.05	—	1.00
	D	0.75	0.20	0.00	—

For each subject's data, either show that $\langle \Omega_g, \succsim_i \rangle$ is a weak order, or indicate which property of a weak order is violated and show an example of the violation. You may assume reflexivity throughout—that is, that $X \succsim_i X \; \forall \; X$ and i. Additionally, if $\langle \Omega_g, \succsim_i \rangle$ turns out to be a weak order for one or both subjects, do the following for such subjects. Assign scale numbers $\phi_i(X)$ from the set {1,2,3,4} for every X in Ω_g so that the numbers $\phi_i(X)$ "represent the order" for subject i in the sense of the "existence part" of the "fundamental weak order representation theorem" we saw in class. Show your scale. Now consider three possible "transformations" of your scale ϕ_i: $u_i(X) = [\phi_i(X)]^2$, $v_i(X) = -3\phi_i(X)$ and $w_i(X) = \{ 2 \text{ if } \phi_i(X) \leq 2 \text{ , } 3 \text{ if } \phi_i(X) \geq 3 \}$. Show that the first transformation still represents the weak order, but that the second and third do not. In those latter cases, explain what is "wrong" with those transformations by referring to the "uniqueness part" of the ""fundamental weak order representation theorem" we saw in class.

(2) Suppose that a consumer values bundles of goods of the form $X = (x_1, x_2, \ldots x_j, \ldots x_k, \ldots, x_n)$, where the x_i are real numbers representing the amount of good i in the bundle. Suppose the consumers' preferences are such that there is a pair of goods j and k for which the consumer cannot tell small differences in consumption of those two goods apart—provided that one of the bundles does not dominate the other (a definition of "dominates" will follow). Mathematically, that is, consider two bundles X and Y which are identical except for the amounts of goods j and k that they contain:

$X = (z_1, z_2, \ldots x_j, \ldots x_k, \ldots, z_n)$, and $Y = (z_1, z_2, \ldots y_j, \ldots y_k, \ldots, z_n)$, where $x_j \neq y_j$ and $x_k \neq y_k$.

Suppose that the following two things are known about the consumer's preferences between bundles like this (other things might be true too, but these two properties are known):

(i) If one bundle dominates the other—that is, contains at least as much of both goods j and k and strictly more of either j or k or both), the "dominating" bundle is strictly preferred to the "dominated" bundle. In other words, the consumer can always apply a "more is preferred to less" rule, regardless of how similar the two bundles are.

(ii) However, in cases where there is no “dominating” bundle, there is always indifference between the two bundles when differences get small enough. Let there be strictly positive constants δ_j and δ_k—what decision theorists would call “just noticeable differences" or JNDs. Suppose it is also known that in cases where there is no dominating bundle:

If $|x_j - y_j| \le \delta_j$ and $|x_k - y_k| \le \delta_k$, then $X \sim Y$.

Please note that this is a one way implication; that is, there may well be cases where $|x_j - y_j| > \delta_j$ and $|x_j - y_j| > \delta_j$ and $X \sim Y$ (I am telling you this because students have given faulty proofs of the following statement by not realizing that the implication above is a one-way implication).

Prove that an ordering of bundles of goods with these two properties is not a weak order, and conclude that it is not possible to represent this ordering by a utility function. Hint: Construct a set of alternatives for which the indifference relation is intransitive. It will be helpful to draw pictures that show the region of bundles which are indifferent to a given bundle, and that dominate a given bundle, in the x_j,x_k space according to the two rules described above. This should help to get your intuition working.

(3) Below I describe several groups of two or more decision sets faced by experimental subjects in decision experiments involving pairs of two-outcome or “binary” lotteries of the form (x,p,y), denoting probabilities p and $1-p$ of receiving x dollars and y dollars, respectively. For each set, I indicate the fraction of the subjects who picked the left-hand lottery in the set. In each group, interpret the data under what people call a “representative decision maker" assumption (that all of the subjects essentially behave like a single decision maker, and so the fraction of subjects choosing the left lottery may be interpreted as this “decision maker's” probability of choosing the left lottery) and decide whether the choices of this “representative decision maker” are consistent with expected utility theory. If not, explain what axiom(s) of expected utility is (are) violated by the data.

Group A.	Set A1: (10,0.55,0) or (9.5,0.60,0)	75% chose the left lottery
	Set A2: (9.5,0.60,0) or (9,0.65,0)	70% chose the left lottery
	Set A3: (9,0.65,0) or (10,0.55,0)	65% chose the left lottery
Group B.	Set B1: (10,0.80,0) or (20,0.40,0)	70% chose the left lottery
	Set B2: (10,0.08,0) or (20,0.04,0)	30% chose the left lottery

In the next two groups, the notation $[(x,p,y),q,z]$ denotes a “two-stage” lottery: In the “first stage,” there is a $1-q$ probability of receiving z dollars immediately, in which case the second stage is irrelevant to payoffs. But there is also a q probability that no money is received in the first stage, but the subject immediately plays out the second stage lottery (x,p,y) (that notation has the same meaning it did for groups A and B above). You may assume that the uncertain events that determine outcomes in the first and second stages are statistically independent. Also assume that, in the experiment, two stage lotteries are represented as a two stage procedure to subjects. That is, when a set given below includes two-stage lotteries, do not assume that that set was presented to subjects in a distributionally equivalent single-stage form.

Group C:	Set C1:	[(10,0.80,0),0.10,0] or [(20,0.40,0),0.10,0]	70% chose the left lottery
	Set C2:	(10,0.80,0) or (20,0.40,0)	70% chose the left lottery
Group D:	Set D1:	[(10,0.80,0),0.10,0] or [(20,0.40,0),0.10,0]	70% chose the left lottery
	Set D2:	(10,0.08,0) or (20,0.04,0)	30% chose the left lottery

Now, think about all of the data from groups B, C and D as coming from a single experiment. That is, imagine a final group as follows:

Group E.	Set E1:	(10,0.80,0) or (20,0.40,0)	70% chose the left lottery
	Set E2:	(10,0.08,0) or (20,0.04,0)	30% chose the left lottery
	Set E3:	[(10,0.80,0),0.10,0] or [(20,0.40,0),0.10,0]	70% chose the left lottery

What axiom(s) of expected utility (if any) are violated by this data?