Dr. Wilcox. Micro I. Problem Set #6.

(1) Suppose that w and r are given labor wage and capital rental rates, respectively, and that Q is a specified level of output that a competitive firm wants to produce at minimum cost. The firm uses only labor L and capital K to produce its output. Someone proposes to use the system

$L = rQ/(w+r)$ and $K = wQ/(w+r)$

as a set of conditional input demand functions for this profit maximizing firm. Does this system obey all of the properties of cost-minimizing conditional input demands? What "cost function" do they together imply? Does this implied cost function satisfy Shepherd's Lemma?

(2) Consider the following expenditure function $E(\boldsymbol{p},u)$ for Bob, where $u > 0$ is a utility level, $\boldsymbol{p}$ is a vector of n strictly positive prices p_i of n quantities of goods x_i ($i = 1, 2, 3,..., n$) consumed by Bob, and the a_i are n nonzero constants:

$$E(\boldsymbol{p},u) = u \cdot \min_i \left(\frac{p_i}{a_i} \right)$$

(a) Verify that this is, in fact, an expenditure function for a rational Bob (homogenous of degree 1 in prices, increasing in u, nondecreasing in p, and weakly concave in $\boldsymbol{p}$) under suitable restrictions on the n constants a_i (state the restrictions).

(b) Use Shepherd's Lemma to get Bob's Hicksian Demand functions. Be careful to state the condition under which any Hicksian demand will be zero versus positive (if any).

(c) Derive Bob's indirect utility function and use Roy's identity to get Bob's Marshallian Demand functions. Be careful to state the condition under which any Marshallian demand will be zero versus positive (if any).

(d) Show that your answers in parts (b) and (c) satisfy the Slutsky Equation.

(e) What is Bob's utility function? (Hint: Reason this out by analyzing the properties of the demand functions--particularly, the conditions under which the demands for goods are zero--and think about what kinds of indifference curves between any two goods cause that…graphical reasoning is fine for this question).

(3) Consider the following expenditure function E($\boldsymbol{p}$,u) for Charlie, where $u > 0$ is a utility level, $\boldsymbol{p}$ is a vector of n strictly positive prices p_i of n quantities of goods x_i ($i = 1, 2, 3,..., n$) consumed by Bob, and the a_i are n nonzero constants:

$$E(\boldsymbol{p},u) = u \cdot \sum_{i=1}^{n} \frac{p_i}{a_i}$$

(a) Verify that this is, in fact, an expenditure function for a rational Charlie (homogenous of degree 1 in prices, increasing in u, nondecreasing in p, and weakly concave in $\boldsymbol{p}$) under suitable restrictions on the n constants a_i (state the restrictions).

(b) Use Shepherd's Lemma to get Charlie's Hicksian Demand functions. Be careful to state the condition under which any Hicksian demand will be zero versus positive (if any).

(c) Derive Charlie's indirect utility function and use Roy's identity to get Charlie's Marshallian Demand functions. Be careful to state the condition under which any Marshallian demand will be zero versus positive (if any).

(d) Show that your answers in parts (b) and (c) satisfy the Slutsky Equation.

(e) What is Charlie's utility function? (Hint: This time, reason this out by looking at the Hicksian demands, and asking yourself what kind of indifference curves between pairs of goods would generate these kinds of substitution effects…graphical reasoning is fine for this question).