Dr. Wilcox. Micro I. Problem set #5.

(1) Suppose that a consumer's preferences for two goods 1 and 2 are representable by the utility function $U(x_1,x_2) = u(x_1) + v(x_2)$, where the functions u and v are continuous with continuous first and second derivatives, strictly increasing and strictly concave. Such a utility function is called an "additively separable utility function." Suppose the consumer has a money income of M and the prices of goods 1 and 2 are p_1 and p_2, respectively. Assume that the consumer values no other goods. Prove the following claims by setting up the LaGrangian for this problem and applying the formal comparative statics recipe to the FOCs. First show that the SOC is satisfied. You may use Cramer's rule if you wish, but you do not have to. However, do <u>not</u> use the Slutsky Equation to get from (a) to (b).

(a) The demands for both goods 1 and 2 are increasing in M (both goods must be normal goods.)

(b) The demand for good 1 (good 2) is decreasing in p_1 (p_2) (both of the optimal demands slope down in their "own price").

(c) Consider a case where good 1 and good 2 are yams in years 1 and 2, respectively; and return to problem (g) of Problem Set #3. How would you express x_1, x_2, M, p_1 and p_2 in terms of the notation you used in that problem (assume $i_s = i_b = i$)? It is very common in "intertemporal choice problems" like this one to assume an additively separable utility function like the one you have just analyzed in general, with the specific form $v(x_2) = \delta u(x_2)$; that is, that the utility of the consumption good (yams) is the same in both years, except for a "discount factor" $\delta \in (0,1)$; so go ahead and assume this specific additively separable form with u strictly increasing and concave: $U(x_1,x_2) = u(x_1) + \delta u(x_2)$. Characterize the solution with appropriate FOCs.

In terms of effects on optimal demands, is the effect of a change in m_1 pretty much like a change in M in the general case you did in parts (a) and (b) above (that is, except for at most a linear multiplier based on some other parameters)? How about a change in m_2? Is a change in π like a standard Marshallian price effect (that is, like a change in p_2 in the general problem you just analyzed, except for perhaps at most a linear multiplier based on some other parameters)? Finally, what is the effect of a change in i on the optimal demand for both year 1 and year 2 consumption? Is a change in i like a change in M alone, or like a change in p_2 alone, or is it in fact a combination of these two kinds of basic effects? Can you sign the effect of this change on year 2 consumption? How about if you knew whether the household was a borrower or lender in year 1? (Hint: Drawing a graphical depiction of the constraint, and thinking carefully about whether and/or how the constraint either "shifts" and/or "rotates"—and around what point--in response to each parameter change, may get your intuition going for most of these questions and help you interpret the math).

(2) Suppose that a consumer's preferences for two goods 1 and 2 are representable by the utility function $U(x_1,x_2) = (x_1-\phi_1)^{\alpha}(x_2-\phi_2)^{ß}$ for all (x_1,x_2) such that both $x_1 \geq \phi_1$ and $x_2 \geq \phi_2$; if either $x_1 < \phi_1$ or $x_2 < \phi_2$, then $U(x_1,x_2) = -1$. α and ß are positive constants and ϕ_1 and ϕ_2 are nonnegative constants. Suppose the consumer has a money income of M and the prices of goods 1 and 2 are p_1 and p_2, respectively; and assume that $M \geq p_1\phi_1 + p_2\phi_2$.

(a) With the utility function defined as it is, and interpreting U = -1 as "death," what are ϕ_1 and ϕ_2?

(b) With your interpretation in part (a) in mind, interpret the meaning of the quantities $p_1\phi_1$ and $p_2\phi_2$ and $M - p_1\phi_1 - p_2\phi_2$.

(c) Set up the LaGrangian, get the FOCs and show that they imply that the optimal goods demands generate the so-called "linear expenditure system" written as follows:

$$p_1\bar{x}_1 = p_1 f_1 + b_1(M - p_1 c_1 - p_2 d_1)$$

$$p_2\bar{x}_2 = p_2 f_2 + b_2(M - p_1 c_2 - p_2 d_2)$$

…and express the six constants b_i, c_i and d_i, i = 1 and 2, in terms of the constants α, ß, ϕ_1 and ϕ_2 (Note: It is not necessarily the case that all six of these constants are different from each other—I say this to prevent you from being confused if <u>some</u> of the constants b_i, c_i and d_i turn out to be equal across the two equations). Algebra-easing hint: Give an argument as to why the solution to maximizing U is the same as the solution to maximizing $ln(U)$; and then use the transformed utility function $ln(U)$ to build your LaGrangian instead of using U.

(d) Suppose now that $\phi_1 = \phi_2 = 0$. Prove that the income elasticities of the optimal goods demands are both equal to 1 in this case. Is this true when they do not equal zero? Use this to comment on the purpose of introducing positive parameters ϕ_1 and ϕ_2 into the utility function you've been considering.