Dr. Wilcox. Micro I. Problem set #3.

(1) Below I describe several choice situations faced by a decision maker in a way meant to convey the nature of the constraints faced by that decision maker. In each case, do the following:

(i) State the name of the decision maker;

(ii) State which letters stand for parameters;

(iii) State which letters stand for choice variables;

(iv) Express the constraints algebraically as a system of one or more inequalities or equalities as appropriate;

(v) If the constraint(s) involves exactly two choice variables, please sketch the set of points satisfying the constraint(s) on a two dimensional graph with appropriately labeled axes.

Note: Most of these are constraint sets that we will use in some extremum problem later in this course.

(a) Congress must choose to divide a maximum total yearly expenditure of E dollars between total dollars D spent on defense and total dollars N spent on nondefense. The president, not congress, determines E before Congress makes this choice.

(b) Bertha needs to decide how much total daily money income M and total daily nonwork time (sleep, leisure, work inside the home, etc.) L she will have. She has a certain fixed daily nonlabor income A which she cannot do anything about. There are 24 hours in her day, period. For the first eight hours she works each day, she receives a wage of w dollars per hour; she receives a wage of $1.5w$ per hour for every hour beyond the eighth which she works (a government-mandated overtime rate, say); and it is illegal for her to work more than 12 hours per day. w is not under her control.

(c) A <u>sample proportion</u> P_i of some binary outcome variable (say, a choice between two alternatives) is always just an estimate of a <u>true</u> value θ_i of some binomial parameter in the population from which that sample was drawn. Suppose that an econometrician is interested in testing theories about the <u>true</u> values θ_1 and θ_2 (which she does not know) in a population on the basis of two observed sample proportions P_1 and P_2 calculated on the basis of a sample she has from that population. The true values must lie between 0 and 1, of course, since these are probabilities. Suppose that the econometrician has a "likelihood function" $L(P_1,P_2|\theta_1,\theta_2)$ which describes how well any hypothesized true values of θ_1 and θ_2 match her observed data (described by the sample proportions P_1 and P_2). She is interested finding out what choices of values of θ_1 and θ_2 will make this match as close as possible (that is, will maximize L), given that a specific requirement of a certain theory is satisfied. That requirement is that <u>both</u> θ_1 and θ_2 lie on the <u>same</u> side of one-half. Note: Please figure out how to write this dual requirement as a single constraint. Hint: You should consider that whether two negatives or two positives are multiplied together, the result is always nonnegative.

(d) A dictator wants to decide how to devote two inputs *K* and *L* to the production of goods 1 and 2. The total inputs devoted to the production of good 1 are denoted K_1 and L_1, while the total inputs devoted to the production of good 2 are K_2 and L_2. The total amounts of the two inputs available to the dictator cannot be changed and are given by K^T and L^T.

(e) A dictator wants to decide how to divide the total amounts available of goods 1 and 2 between two individuals *a* and *b*. The amounts of the two goods given to *a* are denoted X_1^a and X_2^a while the amounts given to *b* are denoted by X_1^b and X_2^b. The total amounts of the two goods available to the dictator cannot be changed and are given by X_1^T and X_2^T.

(f) A firm wants to figure out what the cheapest way to produce a certain given amount of output *Q* is. The firm must choose a combination of labor *L* and capital *K* to produce the good. There is a "production function" $f(K,L)$ which gives the largest amount of output the firm can get from each combination of *K* and *L* it considers.

(g) A household knows it will receive m_1 dollars of income at the beginning of this year (called year 1) and m_2 dollars of income at the beginning of next year (year 2); and it dies at the end of year 2. It has no control over these amounts of income. However, dollars of year 1 income can be saved and will earn a yearly interest rate of i_s per dollar, payable at the beginning of year 2 along with the number of dollars saved. Also, the household may instead borrow against year 2 income if they wish. If they borrow any dollar for purchases in year 1, they will have to pay back (at the beginning of year 2) those dollars plus an interest rate of i_b per dollar borrowed at the beginning of year 1. The household survives on a single good (let's call it "yams," which are similar to sweet potatoes), in both years, which may be purchased at the beginning of each year. Yams purchased at the beginning of year 1 cannot be stored for year 2. The household must choose what its purchases of yams at the beginning of years 1 and 2, y_1 and y_2 respectively, will be; these purchases must be financed out of the net dollars available to the household at the beginning of each year, on the basis of their available income and the effects of any borrowing or saving decisions they might have made. Yams cost a dollar each at the beginning of year 1; and there is a yam inflation rate of π dollars per year in this economy (so that the year 2 price of yams is $1+\pi$. Do two variations of this situation: First consider the case where the two interest rates are equal, that is $i_s = i_b = i$, and show that the constraints may be expressed as a single inequality. Then consider the case where i_s is strictly less than i_b, where the relevant constraints are a pair of inequalities (why is this?) What does this latter case mean, in practical terms?