Dr. Wilcox. Micro I. Problem Set #10.

(1) Consider households with utility function $U(x_1,x_2) = x_1^{\alpha}x_2^{1-\alpha}$ (the Cobb-Douglas function).

(a) Show that this function is homothetic. (Just restate your proof from problem set #7).

(b) Suppose that the household with the utility function given above faces constant prices $p_1$ and $p_2$ for goods 1 and 2, and let its income be equal to $M$. Derive the Marshallian demands for goods 1 and 2 for this household. By analyzing the demand functions, give a succinct interpretation of the parameter $\alpha$ in these demand functions (i.e., can we say that $\alpha$ is equivalent to some interesting thing for utility-maximizing consumers with this utility function)?

(c) Now, consider an economy where each household $k = 1, 2,..., K$ has its own income $M^k$ and the utility function as given above, except that each household has its own value $\alpha^k$ of the utility function parameter $\alpha$. Write down the aggregate demand for good 1, $X_1(M^1,M^2,M^3,...,M^K,p_1,p_2)$ (just the sum of the individual Marshallian demands).

(d) Can the aggregate demand for good 1 be written in the form $f(M^T,p_1,p_2)$, where $M^T$ is the sum of the incomes of all $K$ households, and the function $f$ doesn't depend on the distribution of $M^T$ amongst the $K$ households? If so, display such a function. If not, conclude that homotheticity alone is an insufficient condition for exact linear aggregation.

(2) (The idea for this problem set comes from Dr. Ben-Gad, formerly of our department). Let there be an economy with just two agents. Both agents have exactly the same utility function, which is given by $U(x_1,x_2) = x_1^{0.5} + ln(x_2)$.

(a) Prove that this utility function is not homothetic.

(b) Let $M$ be the income of either agent, and let $p_1$ and $p_2$ be the prices of goods 1 and 2. Derive the Marshallian demand for $x_1$. Show that this demand function is convex in $M$.

(c) Now suppose that one agent has income $M-\varepsilon$, while the other has income $M+\varepsilon$. You can think of $\varepsilon$ as being a government transfer from the first agent to the second agent, where both agents start with the same income $M$. Write down the aggregate demand for good 1, $X_1(M-\varepsilon,M+\varepsilon,p_1,p_2)$. Show that this function cannot be expressed in the form $f(2M,p_1,p_2)$ (that form would imply that aggregate demand is independent of the transfer amount $\varepsilon$), and conclude that identical utility functions across agents, in and of itself, is also an insufficient condition for exact linear aggregation.

(d) What value of $\varepsilon$ would minimize $X_1$ (for given values of $M$, $p_1$ and $p_2$) in this particular economy? Without actually solving for the Marshallian demands, but rather simply knowing that both agents' budget constraints bind at their utility maximum, give an argument that the aggregate demand for good 2 is maximized at the same value of $\varepsilon$.

(3) Consider the utility function $\alpha ln(x_1) + (1-\alpha)ln(x_2-\phi)$, which is a special case of the "Stone-Geary" utility function you saw in problem set 5 (the one that yields the "linear expenditure system"). Let us suppose throughout this new problem that $\alpha$ and $\phi$ do not vary over households--they are just given parameters for each and every household. Remember that the parameter $\phi$ is interpretable as a minimum "subsistence" amount of the second good (assume that $\phi > 0$).

Suppose that good $x_1$ is the good "leisure," and that good $x_2$ is actually "consumption," defined as dollars spent on goods. The price of leisure is the wage rate $w^k$ (since you have to give up an hour of income to "purchase" an hour of leisure) and the price of consumption is obviously 1. Also let each household's "full income" (labor economists sometimes call it this) for "purchases" of leisure and consumption goods be equal to $Tw^k$, where $T$ is the number of hours available to the household for division between work and leisure.

(a) Derive each household $k$'s Marshallian demand functions for leisure and consumption. You may assume that $w^K$ is large enough (for every household $k$), so that an interior optimum exists with $x_1 > 0$ and $x_2 > \phi$ for every household. Treat full income $Tw^k$ as you normally would income $M$ in deriving these functions, so that your demand functions will be expressed as functions of full income $Tw^k$ and the one price (the wage rate $w^k$, where it appears without $T$ multiplying it). What is each household's labor supply function? Show that an increasing share of each household's full income will be spent on leisure as full income increases.

(b) Are the labor supply functions of these households aggregable in the sense that aggregate labor supply can be written as a function of $T\sum w^k$ (aggregate full income) and/or the mean wage rate $W = \sum w^k/K$, without paying any attention in that function to the distribution of wage rates?

(c) Now transform the <u>labor supply</u> functions into <u>value of labor supply</u> or <u>labor earnings</u> functions by multiplying household $k$'s labor supply function by its wage rate $\mathrm{w}^{\mathrm{k}}$. Define the aggregate labor earnings function as the sum of each household's labor earnings function. Can this aggregate earnings function be written as a function of $T\sum w^k$ (aggregate full income) and/or the mean wage rate $W = \sum w^k/K$, without paying any attention in that function to the distribution of wage rates?