

Dr. Wilcox. Micro I. Problem Set #1.

(1) Suppose that a competitive wild rice industry produces wild rice under the following conditions. No wild rice can be gathered for less than 2 dollars per pound (\$2/lb.); that is, the marginal cost of the first pound gathered is \$2/lb. Thereafter additional wild rice can be gathered at the marginal cost of foraging activity, which rises continuously as increasing amounts of foraging activity take place. In fact for every additional 50,000 lbs. of wild rice gathered, the marginal cost of additional foraged wild rice will rise at a continuous, constant rate of \$1 per 50,000 lbs. For instance the marginal cost of the 15,000th lb. of wild rice gathered would be \$2.30, and the marginal cost of the 300,000th lb. of wild rice would be \$8. However, identical wild rice can be cultivated, instead of gathered by foraging, at a constant marginal cost of \$10/lb. regardless of the number of lbs. that are cultivated. The demand for wild rice is given by $q_d = 3,400,000 - 200,000p_d$.

Express the supply curve of wild rice algebraically and graphically. Suppose the government is contemplating an excise tax t on wild rice. Express government revenue as a function of t algebraically and graphically. Show that the function has the following kinds of extrema for choices of $t \in [0,15]$; find the value of t at which each extrema occurs; and find government revenue at each extrema:

- (a) Two local corner minima and one local interior minimum.
- (b) Two global minima.
- (c) Two local interior maxima.
- (d) One global maximum.

(2) Return to the wild rice problem above. Suppose instead that a single monopolist “controls” the aggregate supply curve above (that is, the monopolist “owns” both foraging for wild rice and the cultivation of wild rice), so that this supply curve is equivalent to this monopolist’s marginal cost curve. Find the monopoly profit-maximizing output and price for wild rice and calculate the monopoly profit. Compare the monopolist's profit-maximizing price to the price paid by consumers with the government’s revenue-maximizing tax rate with perfect competition that you discovered in (1) above. Would the relationship between these two prices be as it is here for any aggregate supply curve? Use graphical analyses of both the government's revenue maximization problem and the monopolist's profit maximization problem, with and without a perfectly elastic supply curve, to show that, in general, the government's revenue maximum and the monopolist's profit maximum may be different.

(3) In all of the following problems, a monopolist produces a total output of Q units per year of some product at a total dollar cost given by the function $C(Q)$ per year (in other words, $C(Q)$ is this monopolist's total cost function). The dollar price p that the monopolist must charge in order to sell Q units of output per year is given by the function $P(Q)$ (in other words, $P(Q)$ is the inverse demand function for the monopolist's product per year). The only variable chosen by the monopolist is output, and the monopolist's objective is profit. Assume throughout the problem that $C(0) = 0$ and that $C(Q)$ is continuous, twice differentiable, strictly increasing and convex; and that $P(Q)$ is bounded, continuous, twice differentiable, strictly decreasing and concave. This is the basic setup. For each variation on this setup (A through F below):

- (i) Write down the monopolist's objective function;
- (ii) State which letter in the objective function stands for the choice variable;
- (iii) State which letters in the objective function stand for functions of the choice variable--that is, letters that are neither parameters nor the choice variable;
- (iv) State which letters in the objective function stand for neither the choice variable nor functions of the choice variable, but rather are parameters beyond the control of the monopolist;
- (v) Write down the first- and second-order conditions for a local interior maximum of the objective function;
- (vi) Sign the terms of the second-order condition and decide whether the second-order condition is satisfied; and
- (vii) Do anything else that you are asked to do for each specific variation.

The variations:

- A. No variations--just the basic setup.
- B. The government takes a fraction $\rho \in [0,1]$ of the monopolist's profit each year (that is, there is a pure profit tax rate of ρ). How does the first-order condition for this variation compare to that in part A? Please interpret this comparison from a behavioral viewpoint.
- C. The government requires that, during any year that the monopolist produces any output, she must pay a license fee F that is independent of the amount of output she sells. How does the first-order condition for this variation compare to that in parts A and B? Please interpret this comparison from a behavioral viewpoint. Are there ANY differences between the monopolist's behavior in these three cases?
- D. The government requires that the monopolist pay t dollars every time it sells a unit of its product (that is, the government imposes an excise tax of t dollars per unit sold by the monopolist.)
- E. The government allows the monopolist to keep any dollar price p_s per unit she likes, but requires that the monopolist collect a dollar price $p_d = (1+t)p_s$ per unit from buyers and turn over tp_s dollars to the government for every unit sold. Put differently, the government sets a sales tax of t dollars per dollar kept by sellers or, equivalently, keeps a fraction $t/(1+t)$ of whatever dollar price p_d the buyers actually pay for the good.

F. Throughout these problems, $C(Q)$ was the cost of production of Q total units of salable output; that is, units produced were always free of defects and hence could be sold. Suppose instead that a gross output of Q_g total units of the product is produced at a cost given by a new function $G(Q_g)$ which is continuous, twice differentiable, strictly increasing and convex and that $G(0) = 0$. Only a fraction $\gamma \in [0,1]$ of the gross output Q_g is salable (that is, free of defect), whereas a fraction $1-\gamma$ is defective and cannot be sold at any price. That is, salable output Q is equal to γQ_g , and Q_g costs $G(Q_g)$. Show how to express the cost of salable output $C(Q)$ in terms of Q_g , γ and the function G ; then do the rest of this variation with the cost of salable output expressed that way.